

Formation of subhorizon black holes from preheating

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We study the production of primordial black holes (PBHs) during the preheating stage that follows a chaotic inflationary phase. The scalar fields present in the process are evolved numerically using a modified version of the HLATTICE code. From the output of the numerical simulation, we compute the probability distribution of curvature fluctuations, paying particular attention to sub-horizon scales. We find that in some specific models these modes grow to large amplitudes developing highly non-Gaussian probability distributions. We then calculate PBH abundances using the standard Press-Schechter criterion and find that overproduction of PBHs is likely in some regions of the chaotic preheating parameter space.

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I. INTRODUCTION

During primordial inflation, spacetime expands exponentially for about 60 e -folds, producing the homogeneous, isotropic and almost flat Universe we observe today. Small fluctuations of the inflationary field are stretched to scales larger than the cosmological horizon and reenter in subsequent epochs to source the cosmic structures. After inflation ends, the energy stored in the dominant field must decay into relativistic particles to create the radiation-dominated environment required by nucleosynthesis, a transition phase referred to as reheating. Modelling reheating remains a challenge since one must deal with the evolution of highly inhomogeneous fields in an expanding background, including nonlinear phenomena up to the time of thermalization of the Universe.

Of particular interest in recent years has been a reheating model in which resonant amplification by the inflaton field leads to particle production, a process known as preheating [1]. Here a spectator field χ is nonminimally coupled with the dominant inflaton ϕ , which oscillates at the bottom of the potential. The quantum fluctuations of χ experience a resonant amplification, causing an exponential growth of its occupation numbers and an explosive production of relativistic particles. The parametric-resonant mechanism of preheating has proved to be extremely efficient for a range of parameter values [2]. To improve our understanding of preheating mod-

els we can study the gravitational instability that will amplify the matter fluctuations. Large matter overdensities may form due to the highly nonlinear physics of the preheating mechanism [3, 4], and the highest density concentrations may collapse and form black holes. Consequently, the observable constraints on the abundance of these primordial black holes (PBHs) could help us to constrain models of preheating. The goal of this paper is to determine the production rate of PBHs during the preheating after a chaotic inflation stage. In this preheating model, the inflaton couples nonminimally to a massless spectator field through a “four-legs interaction” term. The evolution of the scalar fields is calculated numerically with a simplified version of the HLATTICE code [5], which performs a three-dimensional integration of the equations of motion for the full nonlinear variables. We find that the amplitude of fluctuations increases rapidly inside the Hubble horizon even when the background energy component behaves almost like radiation. At the same time, the probability distribution of curvature inhomogeneities develops a skewed profile. We provide an example where both effects conspire to produce a considerable number of PBHs at subhorizon scales (a possibility explored in Ref. [6]).

Our paper is organized as follows. The following section presents the elements of the preheating model we work with, including a description of the numerical setup in the present study. Section III describes the criteria used to account for the formation of PBHs from overdensities at subhorizon scales in the preheating phase. In Sec. IV we present the probability density distribution for inhomogeneities in our model and estimate the probability of PBH formation. We conclude in Sec. V with a summary and a discussion of the possible extensions to our study.

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II. PREHEATING MODEL

The preheating model we consider is given by the Lagrangian

$$\mathcal{L} = \frac{1}{2}\phi_{,\alpha}\phi^{,\alpha} + \frac{1}{2}\chi_{,\alpha}\chi^{,\alpha} - \frac{1}{2}m^2\phi^2 - \frac{1}{2}g^2\phi^2\chi^2, \quad (1)$$

where $\phi(\mathbf{x}, t)$ is the inflaton, and $\chi(\mathbf{x}, t)$ is the auxiliary, spectator field; both are non-linear functions of time and space. The index α denotes the spacetime coordinates $(0, 1, 2, 3)$, $m = 10^{-6}m_{\text{Pl}}$ is the inflaton mass, and g^2 is the coupling constant, which is a free parameter in our study.

To study the evolution of the matter fields together with that of the spacetime, we have used a modified version of the HLATTICE code [5], with a flat Friedmann-Lemaître-Robertson-Walker metric, for which the dynamics is governed by the homogeneous scale factor $a(t)$ only. Note that this implies the suppression of spacetime fluctuations in our simulations, but it can be regarded as the choice of a flat gauge in the context of cosmological perturbation theory [7]. Each hypersurface of constant time is thus conformal to flat space (we shall discuss this point further in Sec. III).

Our numerical simulation starts from about one e -fold before the end of inflation (defined here as the time when the Hubble scale finds a global minimum). We choose background values for the fields at the initial time as $\phi_{\text{init}} = \bar{\phi} = 0.3M_{\text{Pl}}$ and $\chi_{\text{init}} = \bar{\chi} = 0$, with a Gaussian distribution of perturbations around these mean values, defined in Fourier space as $|f_k|^2 = 1/(2\omega_k)$ and $|\dot{f}_k|^2 = \omega_k/2$. Here $\omega_k = \sqrt{k^2 + m_f^2}$ for each one of the fields $f : (\phi, \chi)$. From the Lagrangian in Eq. (1) we can read the squared effective masses m_f^2 of the fields as $m_\phi^2 = m^2 + g^2\chi^2$ and $m_\chi^2 = g^2\phi^2$. In particular we will test for PBH formation considering two values of g^2 commonly used in preheating studies: **case I** takes $g^2 = 6.5 \times 10^{-8}$, and in **case II**, we consider $g^2 = 2.5 \times 10^{-7}$ (more details of the simulations can be found in Refs. [5, 8, 9]).

The growth of the modes χ_k in Fourier space yields an increase in the number of created particles. Indeed, the number density n_k of particles with momentum \mathbf{k} can be evaluated as the energy of that mode divided by the energy of each particle, obtaining

$$n_k = \frac{\omega_k}{2} \left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2 \right) - \frac{1}{2}, \quad (2)$$

and, following Ref. [5], we approximate $\omega_k \simeq k$ to avoid ambiguities in the nonlinear regime.

Figure 1 shows the exponential increase in n_k for modes in the resonance band with the characteristic step-like profile observed in previous works (e.g. Ref.[2]). We confirm the exponential growth of the occupation number but in a dynamic background, an improvement over Ref. [2], which follows the evolution of scalar field in a spacetime dominated by dust (c.f. Fig. 4 of Ref. [2]). As

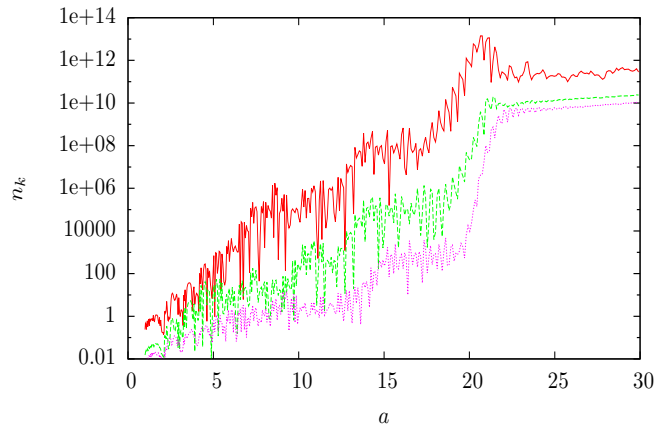


FIG. 1. Time evolution of occupation number n_k as defined in Eq. (2). The large value of n_k justifies the classic treatment of inhomogeneities in our problem. The top line represent a mode that started outside of the horizon, and middle and lower lines refer to modes that remained inside the horizon at the end of inflation.

indicated in Ref. [10], n_k helps us to understand the energy distribution for each mode. Once the distribution hits the highest k mode resolved by the simulation, the energy is reflected back to the infrared modes. For the model and resolution considered here, the numerical simulation can be trusted up to a final time t_f where $a(t_f) = 30$ (well within the broad parametric resonance stage), just before the energy reflection effect kicks in and develops spurious modes.

Because of the exponential growth in the occupation number shown in Fig. 1, we can treat the energy density in classical terms nearly after the end of inflation. Thus, we are free to study the formation of PBHs following the usual methods. The total energy density and pressure are defined by

$$\begin{aligned} \rho(\mathbf{x}, t) &= \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}(\nabla\chi)^2 + V(\phi, \chi) \\ p(\mathbf{x}, t) &= \frac{1}{2}\dot{\phi}^2 - \frac{1}{6}(\nabla\phi)^2 + \frac{1}{2}\dot{\chi}^2 - \frac{1}{6}(\nabla\chi)^2 - V(\phi, \chi), \end{aligned} \quad (3)$$

and can be split in a homogeneous part and inhomogeneous fluctuations. The homogeneous part of the energy density and pressure, $\bar{\rho}(t)$ and $\bar{p}(t)$, are defined through their spatial average at every time step during the simulations. Instead of plotting the behavior of these homogeneous quantities, we follow the evolution of the time dependent equation of state $w(t) \equiv \bar{p}/\bar{\rho}$, which plays a significant role on both the analysis of matter-radiation transition and in determining the criterion of gravitational collapse of overdense regions into PBHs.

The oscillations of the inflaton about the minimum of its quadratic potential are translated into oscillations of w , with time average $\langle w \rangle \approx 0$ at the beginning, and, by the energy transference to the (massless) spectator field, $0 < \langle w \rangle \lesssim 1/3$ at the end of the simulation, Note that

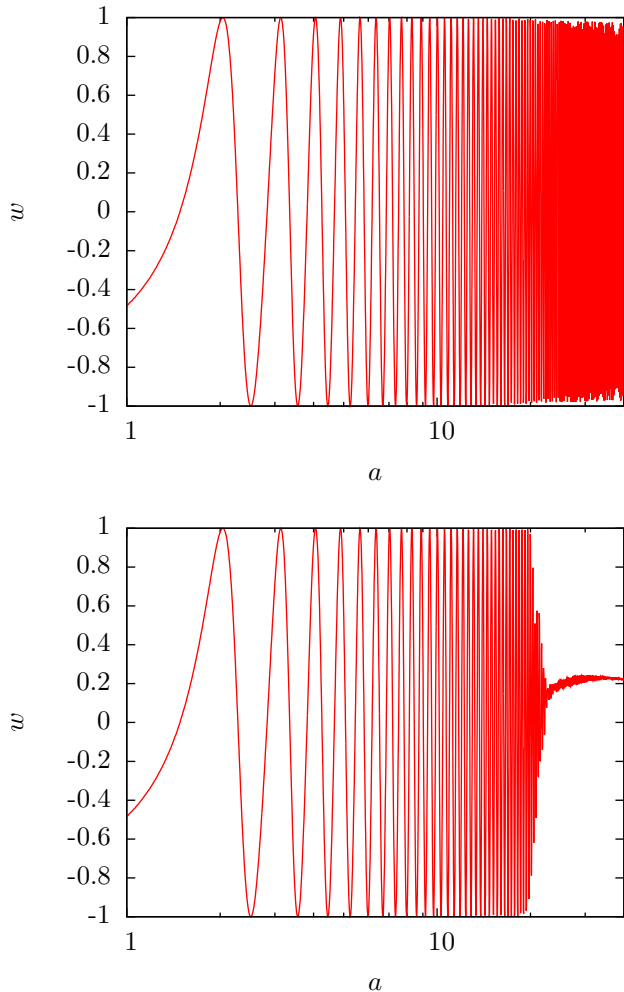


FIG. 2. Time evolution of the equation of state w for case I (top) and case II (bottom). Only for the latter w steadily converges toward a constant value. See the text for more details.

the complete radiation domination can be achieved only in an ideal preheating model.

To transfer energy more effectively and to produce higher redispersion effects between the k modes of both fields, which eventually break the oscillations of the background quantities and stabilize the equation of state, we could select a higher value for the coupling constant g^2 . But, due to the high nonlinearity of the process, that naive selection does not necessarily improve the preheating process.

Figure 2 shows that the stabilization of w at the end of preheating is not generic in the model (1), and neither case I nor Case II are able to produce an ideal complete preheating, even though case II produces a $w \approx cte = 0.2$ (more general analyses about the effectiveness of the model (1) are reported in Ref. [10]).

III. PBH FORMATION CRITERIA

Primordial black holes form from overdense regions in high density environments. Fluctuations with large density contrast $\delta = \delta\rho/\bar{\rho}$ overcome the pressure of the environment and detach from the background expansion. In the standard scenario, if the amplitude of an overdensity of size L is large enough, it will recollapse as soon as it enters in causal contact, that is, when $L_H = 1/H$. As a result of the gravitational collapse, a black hole of mass comparable to the Hubble mass is formed right after horizon crossing. The threshold amplitude for the density contrast δ_{th} to reach the collapse was first estimated to be $\delta_{th} \simeq w$ in a barotropic fluid with pressure $p = w\rho$ using the Jeans criterion for overdensities in a Friedmann background [11]. Recently, simulations considering inhomogeneous cosmologies have determined a threshold amplitude for the comoving matter overdensity as $\delta_{th} \approx 0.41$ in a pure radiation background [12]. Extensions to general barotropic fluid backgrounds are considered in Ref. [13].

The threshold for collapse is best expressed in terms of the gauge-invariant curvature perturbation ζ [14]. This is defined in the uniform density gauge as

$$\zeta = -\psi - H \frac{\delta\rho}{\dot{\rho}}. \quad (4)$$

For the cases that concern us, we note from Fig. 3 that during preheating the fluctuations grow in amplitude inside the cosmological horizon (as we shall see in detail in Sec. IV). This indicates that PBHs are more likely to form inside the cosmological horizon, and a different criterion is required to set the threshold of amplitudes that collapse and form PBHs. The formation of PBHs at scales inside the Hubble horizon has been largely ignored due to the linear Jeans instability criterion, which for a radiation-dominated universe sets the scale of instability close to the Hubble scale. Furthermore, in the dustlike environment, typical of phase transitions in unification theories, PBHs are thought to form at small scales and from overdensities of arbitrarily small amplitude because there is no pressure to prevent the collapse [15]. However, scattering of small black holes could prevent the formation of larger PBHs, e.g., Ref. [16]. In Ref. [6], the formation of PBHs at subhorizon scales is studied. The authors show that the threshold amplitude for the metric fluctuation to form a black hole at scales well inside the horizon can be taken to be equivalent to the known $\zeta_{th} = 0.7$ of the radiation background.¹ Starting from the matter overdensity in the flat gauge δ_{flat} , we can compute

¹ The threshold value given in Ref. [6] (see also Ref. [17]) and used in this work is a conservative estimate. The precise determination of the sub-horizon threshold value is beyond the scope of this paper.

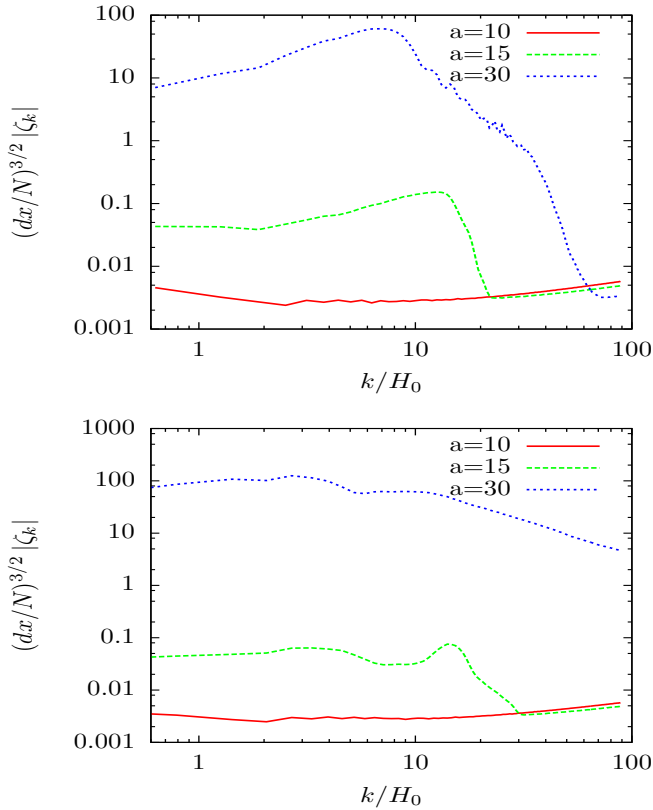


FIG. 3. Time evolution of the mean amplitude of $\zeta_k(t)$ as a function of the wave number k . Note the development of a maximum for a particular scale is formed inside the initial cosmological horizon scale $L_{H0} = 1/H_0$.

the curvature fluctuation as Eq. (4),

$$\zeta = -H \frac{\delta \rho_{\text{flat}}}{\dot{\rho}} = \frac{1}{3} \frac{\delta_{\text{th}}}{1+w}. \quad (5)$$

A key result in our simulation is presented in Fig. 3, where we plot the evolution of the modes $\zeta_k(t)$ over the range of comoving scales covered by the simulation box; more precisely, we plot the evolution of the combination $(dx/N)^{3/2} |\zeta_k|$, which is a box size independent quantity. This results show that the curvature perturbation grows and tends to peak at scales below the Hubble horizon; this motivates the consideration of PBH formation at subhorizon scales.

In the following, we use the threshold value $\zeta_{\text{th}} = 0.7$ to select those overdensities that will inevitably form black holes at subhorizon scales and look into the abundance of PBHs.

It is important to stress that all the analytical and numerical threshold estimates have been found for cosmological phases characterized by a *constant* w , and this validates our threshold definition at the end of preheating for case II, where w is stabilized by the redispersion effects. For case I, the validity of $\zeta_{\text{th}} = 0.7$ is less clear and must be considered with some care.

IV. PROBABILITY OF PBH FORMATION IN PREHEATING

To estimate the mass fraction of black holes at $t = t_f$, $\beta_{\text{PBH}}(M) \equiv \rho_{\text{PBH}}/\rho(t_f) \simeq \rho_{\text{PBH}}/\bar{\rho}(t_f)$, we first construct the probability distribution of the curvature perturbation ζ at a specific scale k [or the equivalent mass $M \propto \bar{\rho}(t_f)k^{-3}$], namely, $\mathbb{P}_M(\zeta)$. As shown in Figs. 4 and 5, the probability distribution function (PDF) develops an appreciable skewness below the Hubble scale L_H .

In general, a non-Gaussian distribution generates non-vanishing moments beyond the variance in real space; likewise, in Fourier space, it generates nonvanishing correlation functions beyond the two-point function. For example, in a typical cosmic microwave background (CMB) anisotropy analysis, the contribution of the reduced bispectrum $f_{\text{NL}}(k_1, k_2, k_3)$ [18] denotes the amplitude of the three-point function. The explicit contribution of f_{NL} to the abundance of PBHs β_{PBH} has been considered by several authors [19–22], showing that the bispectrum can significantly modify the abundance of PBHs with respect to the Gaussian case. This may be used to constrain the non-Gaussian parameters, f_{NL} , g_{NL} , etc., from the observational bounds on the abundance of PBHs. However, a PDF reconstructed from the cosmological non-Gaussianity parameters alone could fail to reproduce the profile of the distribution tail [23], which precisely accounts for the large amplitudes we want to integrate. Because of this, and since we know the solutions in real space, we focus our attention on the PDF that directly arises from the distribution of $\zeta(\mathbf{x})$ in the lattice.

As customarily done, we use the PressSchechter formalism to compute the mass fraction from the non-Gaussian PDF. Applied to the distribution of curvature fluctuations, this yields

$$\beta_{\text{PBH}}(M) = 2 \int_{\zeta_{\text{th}}}^{\infty} \mathbb{P}_M(\zeta) d\zeta. \quad (6)$$

We have computed density profiles $\mathbb{P}(\zeta)$ for different scales by looking at the overdensities of size a few times the resolution scale. In the plots of Fig. 4 for the case I, the mean amplitude of ζ_k shows a negatively skewed distribution for modes inside the horizon, while for case II, the distribution is positively skewed. This leads to an enhancement in PBH formation (with respect to the Gaussian) for the model of case II.

Because of the limited size of our simulation box, the incomplete sample of modes translates into a truncated tail of the numerically generated PDF. To estimate the complete probability of large amplitude fluctuations, we carefully fitted the numerical PDF with analytic distributions which bound the true distribution from above and below (see Fig. 6). For these analytical PDFs, we can easily evaluate the integral of Eq. (6). As a result, we compute upper and lower bounds to the true β_{PBH} . In Table I, we present approximate values of β_{PBH} , calculated at t_f , for overdensities of size of a fraction L/L_H of the horizon scale. The value of β_{PBH} increases with

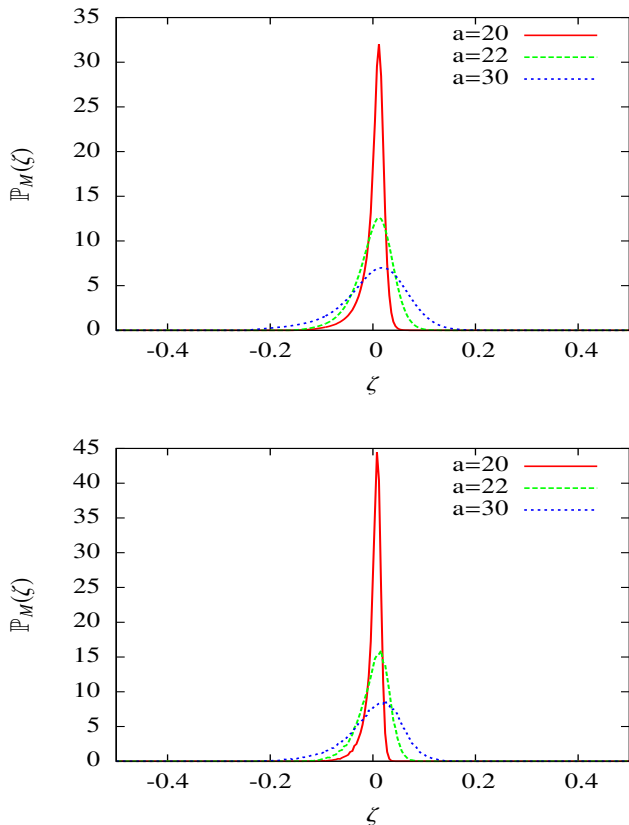


FIG. 4. Probability distribution function of $\zeta_k(t)$ for two modes at different times of evolution in case I. The top (bottom) panel represents the PDF for a mode $L_H(t_f)/L = 46$ ($L_H(t_f)/L = 23$) at the end of the simulation when $a(t_f) = 30$. Note that the shape of the distribution evolves with time through the preheating phase generating negatively skewed distributions.

the wave number as a consequence of the non-Gaussian evolution of fluctuations inside the cosmological horizon. The development of non-Gaussianity of the PDFs is evident from the snapshots of evolution illustrated in Figs. 4 and 5.

Let us finish this section by analyzing the results of Table I in light of the bounds to the PBH mass fraction at the smallest scales. It is known that the Hawking evaporation process of PBHs may halt at the Planck scale. The remnant Planck-mass black holes would survive behaving as a component of the dark matter until the present time. Since their mass fraction cannot exceed that of dark matter, one can impose a bound to their abundance, that is [24],

$$\beta_{\text{PBH}}(M_{\text{PL}}) < 10^{-28} \left(\frac{M}{M_{\text{Pl}}} \right)^{3/2} \quad (7)$$

where M_{Pl} is the Planck mass. When we compute the physical mass enclosed in a region of radius $L/L_H = 1/16$ at t_f , we note that the PBHs formed at that scale would

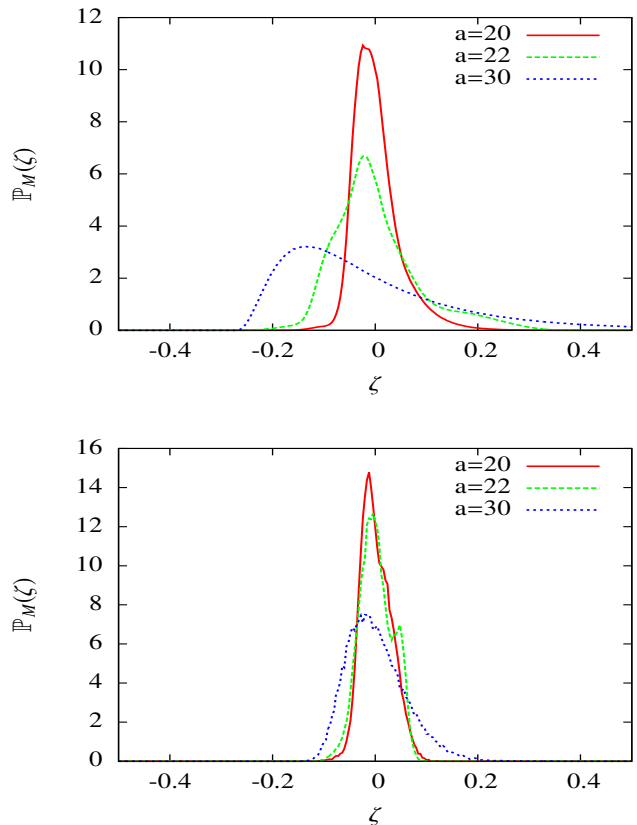


FIG. 5. Same as Fig. 4 but for the model of case II. In this case, the distributions are positively skewed, a feature that favors the formation of PBHs.

have $M_{\text{PBH}} \approx 10^6 M_{\text{Pl}}$. For this scale, we note that the minimum value β_{min} reported in Table I exceeds the bound imposed by Eq. (7). On larger scales, β_{PBH} falls below the observational bounds. As for smaller scales, they lie too close to the resolution scale of our simulation, and thus we do not consider them physical (a complete set of bounds is reported in Ref. [25] and see also Ref. [26] for specific bounds to ζ_k). The implications and limitations of the results derived are discussed in the following last section.

V. DISCUSSION

In this paper, we have considered the possibility of forming sub-Hubble scale black holes, solving numerically the classical governing equations of a model of preheating after chaotic inflation. We have focused on computing the distribution of the curvature fluctuation ζ from the sample of modes in the simulation box of size smaller than the Hubble scale L_H . For the typical values of the coupling parameter $g^2 = 2.5 \times 10^{-7}$ in the reheating model (case II of this paper), we find that the small size inhomogeneities grow in amplitude. When constructing

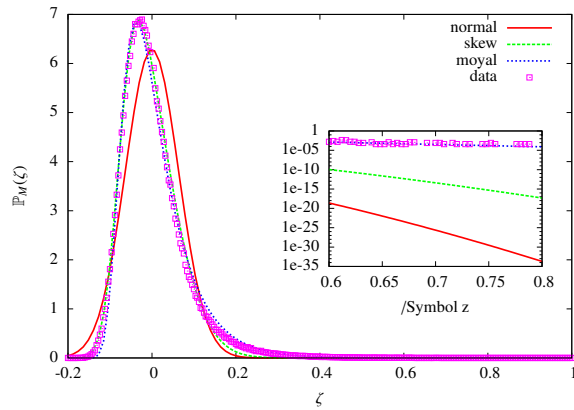


FIG. 6. Tail of the PDF for the scale $L_H(t_f)/L = 23$. The true normalized distribution is approximated by two analytical PDFs. From below, the skewed normal distribution, and from above, the closest approximation is a Moyal distribution, which is the best approximation to the tail of the distribution.

the PDF, we note that large skewness is developed in the probability profile as the modes enter the cosmological horizon (cf. Fig. 5). We find that, at late enough times, primordial black holes of mass about 1 g could be substantially produced, saturating the observational bounds when a Planck mass remnant survives the evaporation process. Interestingly enough, a smaller value of the field coupling $g^2 = 6.5 \times 10^{-8}$ (case I) shows no such overproduction of PBHs. This is because of the development of a negatively skewed PDF as shown in Fig. 4.

One can argue that dependence of the PBH formation rate on the coupling parameter g^2 is due to the existence of resonance bands for the enhancement of fluctuations inside the cosmological horizon [2], and some studies have argued that PBH overproduction is not a generic feature of preheating. Indeed, Ref. [27] finds an insignificant mass fraction β_{PBH} assuming matter fluctuations collapsing at horizon scales and with a Gaussian distribution of probability. Our results show that the Gaussian distribution underestimates the true probability for case II (see the first column of Table I for a β_{PBH} computed from the Gaussian PDF). If we relax the assumptions of Ref. [27] and consider the possibility of PBH formation below the Hubble scale, we find a clearly non-Gaussian PDF from the distribution of fluctuations in the numerical simulation. Our estimations of the PBH mass fraction in Table I indicate that the overproduction of PBH is a likely feature in preheating, and this possibility must be studied in more detail.

In cosmology, PBHs present us with a unique tool to probe the small scales which are not covered by CMB and large scale structure surveys. In the present paper, we show that, while strong couplings between the fields are useful to reheat the Universe efficiently, it is important to test these viable models for PBH overproduction due to the highly nonlinear physics involved in the preheat-

L_H/L	Case	β_{gauss}	β_{max}	β_{min}
46	I	2.94×10^{-31}	2.94×10^{-31}	6.05×10^{-64}
46	II	5.53×10^{-6}	4.7×10^{-3}	1.5×10^{-4}
23	I	4.59×10^{-32}	4.59×10^{-32}	5.83×10^{-67}
23	II	2.26×10^{-16}	2.6×10^{-4}	6.63×10^{-10}
16	I	2.20×10^{-33}	2.20×10^{-33}	1.23×10^{-71}
16	II	1.37×10^{-28}	2.3×10^{-5}	4.52×10^{-16}
9	I	3.19×10^{-37}	3.19×10^{-37}	7.93×10^{-87}
9	II	2.37×10^{-64}	2.6×10^{-7}	5.49×10^{-33}
8	I	5.14×10^{-40}	5.14×10^{-40}	5.87×10^{-94}
8	II	2.58×10^{-80}	5.32×10^{-8}	3.98×10^{-40}
6	I	2.38×10^{-43}	2.38×10^{-43}	5.95×10^{-114}
6	II	7.18×10^{-99}	1.15×10^{-8}	3.39×10^{-49}

TABLE I. β_{PBH} estimations for modes inside the horizon in both cases I and II computed from Eq. (6) with a threshold value $\zeta_{\text{th}} = 0.7$. The statistics of smaller modes presents a considerable uncertainty due to the resolution of our numerical simulation.

ing stage. In this work, we have put forward a method to account for the mass fraction of PBHs, and we shall eventually use it to constrain preheating models.

Motivated by the results presented here, we aim to evaluate a much larger sample of coupling values g^2 in a subsequent work, where we shall also take into account the evolution of metric perturbations and explore a larger range of scales. Another aspect that deserves further study is the criterion used to determine which overdensities will collapse to form PBHs during preheating. The threshold values reported in the literature for scalar fields or barotropic fluids consider mostly collapsing inhomogeneities as soon as they enter the cosmological horizon. The growth of large inhomogeneities due to the nonlinear interactions inside the horizon might eventually require a criterion for PBH formation at subhorizon scales beyond that presented in Ref. [6], in order to constrain the parameter space with even higher accuracy.

We finally note that the overproduction of PBHs could alter the mechanism of transition to a radiation-dominated stage: It is known that preheating in chaotic models cannot drive the Universe to a radiation stage, and one has to consider other higher couplings in the fields to achieve that [10, 28]. According to our results, the overproduction of PBHs during preheating opens the possibility to reconsider PBH evaporation as an auxiliary mechanism to reheat the Universe [29]; such a mechanism may benefit unification models [30].

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- [1] J. H. Traschen and R. H. Brandenberger, Phys.Rev. **D42**, 2491 (1990); Y. Shtanov, J. H. Traschen, and R. H. Brandenberger, Phys.Rev. **D51**, 5438 (1995), arXiv:hep-ph/9407247 [hep-ph]; L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys.Rev.Lett. **73**, 3195 (1994), arXiv:hep-th/9405187 [hep-th].
 - [2] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys.Rev. **D56**, 3258 (1997), arXiv:hep-ph/9704452 [hep-ph].
 - [3] A. M. Green and K. A. Malik, Phys.Rev. **D64**, 021301 (2001), arXiv:hep-ph/0008113 [hep-ph].
 - [4] M. A. Amin, R. Easther, H. Finkel, R. Flauger, and M. P. Hertzberg, Phys.Rev.Lett. **108**, 241302 (2012), arXiv:1106.3335 [astro-ph.CO].
 - [5] Z. Huang, Phys.Rev. **D83**, 123509 (2011), arXiv:1102.0227 [astro-ph.CO].
 - [6] D. Lyth, K. A. Malik, M. Sasaki, and I. Zaballa, JCAP **0601**, 011 (2006), arXiv:astro-ph/0510647 [astro-ph].
 - [7] K. A. Malik and D. Wands, Phys. Reports **475**, 1 (2009), arXiv:0809.4944.
 - [8] G. N. Felder and I. Tkachev, Comput.Phys.Comm. **178**, 929 (2008), arXiv:hep-ph/0011159 [hep-ph].
 - [9] E. Torres-Lomas and L. A. Urena-Lopez, AIP Conf.Proc. **1548**, 238 (2012), arXiv:1308.1268 [astro-ph.CO].
 - [10] D. I. Podolsky, G. N. Felder, L. Kofman, and M. Peloso, Phys.Rev. **D73**, 023501 (2006), arXiv:hep-ph/0507096 [hep-ph].
 - [11] B. J. Carr, Astrophys.J. **201**, 1 (1975).
 - [12] I. Musco, J. C. Miller, and L. Rezzolla, Class.Quant.Grav. **22**, 1405 (2005), arXiv:gr-qc/0412063 [gr-qc].
 - [13] I. Musco and J. C. Miller, Class.Quant.Grav. **30**, 145009 (2013), arXiv:1201.2379 [gr-qc]; T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. **D88**, 084051 (2013), arXiv:1309.4201 [astro-ph.CO].
 - [14] M. Shibata and M. Sasaki, Phys.Rev. **D60**, 084002 (1999), arXiv:gr-qc/9905064 [gr-qc]; A. M. Green, A. R. Liddle, K. A. Malik, and M. Sasaki, Phys.Rev. **D70**, 041502 (2004), arXiv:astro-ph/0403181 [astro-ph].
 - [15] M. Y. Khlopov and A. Polnarev, Phys.Lett. **B97**, 383 (1980).
 - [16] L. Alabidi, K. Kohri, M. Sasaki, and Y. Sendouda, JCAP **1305**, 033 (2013), arXiv:1303.4519 [astro-ph.CO].
 - [17] I. Zaballa, A. M. Green, K. A. Malik, and M. Sasaki, JCAP **0703**, 010 (2007), arXiv:astro-ph/0612379 [astro-ph].
 - [18] D. H. Lyth and A. R. Liddle, (2009).
 - [19] J. C. Hidalgo, (2007), arXiv:0708.3875 [astro-ph].
 - [20] R. Saito, J. Yokoyama, and R. Nagata, JCAP **0806**, 024 (2008), arXiv:0804.3470 [astro-ph].
 - [21] C. T. Byrnes, E. J. Copeland, and A. M. Green, Phys.Rev. **D86**, 043512 (2012), arXiv:1206.4188 [astro-ph.CO].
 - [22] S. Young and C. T. Byrnes, Journal of Cosmology and Astroparticle Physics **2013**, 052 (2013).
 - [23] S. Shandera, A. L. Erickcek, P. Scott, and J. Y. Galarza, Phys.Rev. **D88**, 103506 (2013), arXiv:1211.7361 [astro-ph.CO].
 - [24] B. J. Carr, J. Gilbert, and J. E. Lidsey, Phys.Rev. **D50**, 4853 (1994), arXiv:astro-ph/9405027 [astro-ph].
 - [25] B. Carr, K. Kohri, Y. Sendouda, and J. Yokoyama, Phys.Rev. **D81**, 104019 (2010), arXiv:0912.5297 [astro-ph.CO].
 - [26] A. S. Josan, A. M. Green, and K. A. Malik, Phys.Rev. **D79**, 103520 (2009), arXiv:0903.3184 [astro-ph.CO].
 - [27] T. Suyama, T. Tanaka, B. Bassett, and H. Kudo, Phys.Rev. **D71**, 063507 (2005), arXiv:hep-ph/0410247 [hep-ph].
 - [28] J. F. Dufaux, G. N. Felder, L. Kofman, M. Peloso, and D. Podolsky, JCAP **0607**, 006 (2006), arXiv:hep-ph/0602144 [hep-ph].
 - [29] J. C. Hidalgo, L. A. Urena-Lopez, and A. R. Liddle, Phys.Rev. **D85**, 044055 (2012), arXiv:1107.5669 [astro-ph.CO].
 - [30] A. R. Liddle and L. A. Urena-Lopez, Phys.Rev.Lett. **97**, 161301 (2006), arXiv:astro-ph/0605205 [astro-ph]; A. R. Liddle, C. Pahud, and L. A. Urena-Lopez, Phys.Rev. **D77**, 121301 (2008), arXiv:0804.0869 [astro-ph].